SYNOPSIS

Quantum many body systems in one and two dimensions are known to exhibit behaviours which are radically different from those displayed by their three dimensional counterparts The novel physics generated ranges from the existence of fundamental particles with fractional statistics in two dimensions called anyons , novel order parameters like chirality which violate the discrete symmetries of parity and time-reversal, and the anomalous behaviour of quantum spin chains

In this thesis , we focus on anyons and spin systems In Part I of the thesis we study varied aspects of a system of "free" anyons We obtain a qualitative picture for the ground state wave function and energy of many anyons in a harmonic oscillator potential and the collective properties of the anyon gas at $T = 0$ In Part II , we study quantum antiferromagnets which are also known to behave differently in low dimensions For instance , integral and half integral spin chains have quite different low-energy properties in the case of nearest neighbour Heisenberg antiferromagnetic interactions We address related issues like , the effects of a time reversal violating three spin interaction term in a spin $1/2$ antiferromagnet on non-bipartite two dimensional lattices and the effects of dimerization and frustration in the one dimensional spin-$1/2$ and spin-1 systems

In Chapter 1 , we briefly introduce the concept of anyons with statistics parameter θ and survey some results pertaining to the few body as well as statistical mechanical aspects of anyons In Chapter 2, we first describe a perturbation technique developed to study a system of anyons with statistics θ near the bosons i e , $\theta = 0$ The ground state energies and wave functions of N anyons in a confining harmonic oscillator potential and an external magnetic field is studied using perturbation theory from the fermionic $(\theta = \pi)$ and bosonic $(\theta = 0)$ ends We find that as the statistics parameter changes, there are repeated level crossings in the ground state between states having different angular momenta and that the ground state energy scales as $N^{3/2}$, as the number of anyons increases to infinity We also find that the these properties are independent of the direction of the applied magnetic field We conclude that the ground state wave function is only piecewise continuous , the number of pieces increasing with the number of anyons Chapter 3 deals with the collective properties of a thermodynamic system of anyons at $T = 0$ The anyons are treated as perturbed bosons and have statistics $\theta = \pi/p$ The anyons are treated in a mean field sense, they move in an average statistical magnetic field with a repulsive Coulomb potential between density fluctuations We use the bosonic perturbation theory of Chapter 2 and find that a system of neutral anyons at $T = 0$ has a superfluid phonon mode with a linear dispersion , and charged anyons exhibit the Meissner effect

Chapter 4 contains a brief exposition of quantum spin systems with antiferromagnetic interactions in one and two dimensions   In Chapter 5 , we study the large-U (where U characterizes the on-site electron-electron interaction) limit of the one-band Hubbard model at half-filling on a non-bipartite two dimensional lattice We find that an external magnetic field induces a three spin chiral interaction at order $1/U^2$ We present a model where, at low temperatures , the chiral term is seen to have a larger effect than the Pauli coupling of the electron spins to the magnetic field The ground state is a singlet with a gap , hence the spin susceptibility is zero while the chiral susceptibility is finite and paramagnetic

In Chapter 6, we study the "phase diagram" and some low energy properties of the isotropic antiferromagnetic spin-1/2 and spin-1 chains with dimerization and frustration , i e , an alternation δ of the nearest neighbour exchanges and a next nearest neighbour exchange J_2 We obtain this phase diagram numerically using the density matrix renormalization group For the spin-1/2 case we find the following results   At $\delta = 0$, the system is gapless for all $J_2 < J_{2c}$ and has a gap for $J_2 > J_{2c} = 0\ 2411$ At J_{2c} and for small δ , the gap above the ground state grows as δ with an exponent $0\ 667 \pm 0\ 001$ The system exhibits a gap in its spectrum for all non-zero δ There is a disorder line given by $2J_2 + \delta = 1$ To the left of the line the system has short ranged Neel order and to its right , spiral order At $\delta = 1$, the system is equivalent to two coupled spin-1/2 chains and there is a gap for all values of the interchain coupling

In the spin-1 case, there is a gapless point at $\delta = 0\ 25 \pm 0\ 01$ and $J_2 = 0$ A line emerges from this gapless point and runs across the entire $J_2 - \delta$ plane upto an almost gapless point at $J_2 = 0\ 725 \pm 0\ 01$, $\delta = 0$ The open chain ground state has a near four-fold degeneracy below this line and is unique above it The disorder line obtained in the spin-1/2 case , exists here also with the added feature that this line does not extend all the way to $\delta = 0$ as in the spin-1/2 case but terminates at $\delta = 0\ 136$ These spin-1 results contradict some analytical results which have been derived using continuum field theoretic approaches